~~CS~~135 Final Exam Sample

#1

$$\sum_{k=2}^{54} (2k-5) = 2\sum_{k=2}^{54} k - 5\cdot(54-2+1) =$$

$$= \cancel{2}\cdot \frac{(2+54)\cdot(54-2+1)}{\cancel{2}} - 5\cdot 53 =$$

$$= 56\cdot 53 - 5\cdot 53 = 51\cdot 53 = 2{,}703$$

$$\boxed{\sum_{k=2}^{54} (2k-5) = 2{,}703}$$

or

$$\sum_{k=2}^{54} (2k-5) = \underset{k=2}{(-1)} + \dots + \underset{k=54}{103} = \frac{(-1)+103}{2}\cdot(54-2+1) =$$

$$= \frac{102}{2}\cdot 53 = 51\cdot 53 = 2{,}703.$$

#3

$\{a_n\}$

$a_n = 1 + (-1)^n, \ n \in \mathbb{Z}^+$

$a_1 = 1 + (-1) = 0$

$a_2 = 1 + 1 = 2$

$a_3 = 1 + (-1) = 0$

$a_4 = 1 + 1 = 2$

.....

$$\boxed{\begin{cases} a_1 = 0 \\ a_{n+1} = (a_n + 2) \bmod 2, \ n > 1 \end{cases}}$$

Final Exam Sample

#2 $\mid n^3 - n \quad \forall n \in \mathbb{Z}^+ \cup \{0\}$

Proof (by math. induction): $P(n): \ \mid n^3 - n$

Basis step: $n^3 - n = 0$ when $n = 0$

$\mid 0$ hence $P(0)$ is true.

Inductive step: assume that for any arbitrary fixed $k \geq 0$
$P(k)$ is true, i.e. $\mid k^3 - k$ } I.H.

Let's show that in this case $P(k+1)$ holds:

$$(k+1)^3 - (k+1) = k^3 + 3k^2 + 3k + \cancel{1} - k - \cancel{1} =$$

$$= \underbrace{k^3 - k}_{\text{divisible by 3 by I.H.}} + \underbrace{3(k^2 + k)}_{\text{divisible by 3}}$$

divisible by 3 by a Theorem ~~1~~ from Chapter 4, section 1. (Divisibility and Modular Arithmetic)

Therefore, we showed that $3 \mid (k+1)^3 - (k+1)$, i.e. $P(k+1)$ is true..

This completes inductive step.

By math. induction we proved that $3 \mid n^3 - n \quad \forall n \in \mathbb{Z}^+ \cup \{0\}$

q.e.d.

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#4

a) $R_1 \circ R_2$

- apply R_2 first, then R_1

R_2:	R_1:		$R_1 \circ R_2$:
(b, a)	(a, a)	$\rightarrow$	(b, a)
	(a, b)	$\rightarrow$	(b, b)
	(a, c)	$\rightarrow$	(b, c)

"transitional" element

and so forth...

$$R_1 \circ R_2 = \{(b,a), (b,b), (b,c), (d,d)\}$$

If you decide to use matrices, then do $M_{R_2} \times M_{R_1}$ (switch!)

b) $R_2 \circ R_1$

- apply R_1 first, then R_2

R_1:	R_2:		$R_2 \circ R_1$:
(a, a)	X	$\rightarrow$	X
(a, b)	(b, a)	$\rightarrow$	(a, a)
	(b, b)	$\rightarrow$	(a, b)
	(b, d)	$\rightarrow$	(a, d)

and so forth...

$$R_2 \circ R_1 = \{(a,a), (a,b), (a,d), (c,c), (c,d)\}$$

If you decide to use matrices, then do $M_{R_1} \times M_{R_2}$

CS135

Sample Final Exam

Part 1

#5

symmetric NO antisymmetric No

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

reflexive ✓

equivalence relation:
- reflexive
- symmetric
- transitive

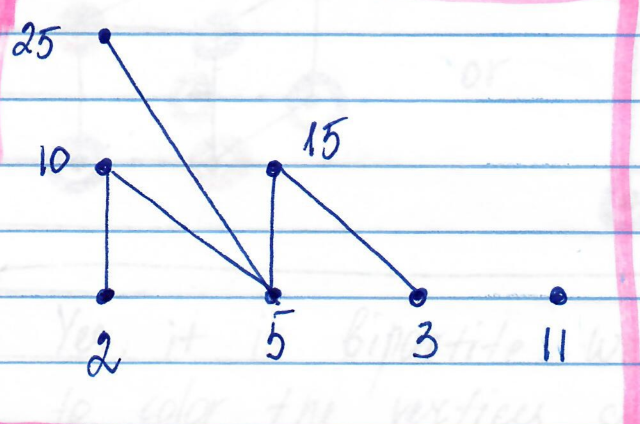
partial order:
- reflexive
- antisymmetric
- transitive

Answer: neither an equivalence relation, nor a partial order.

#6

$\{2, 3, 5, 10, 11, 15, 25 \mid \mid\}$

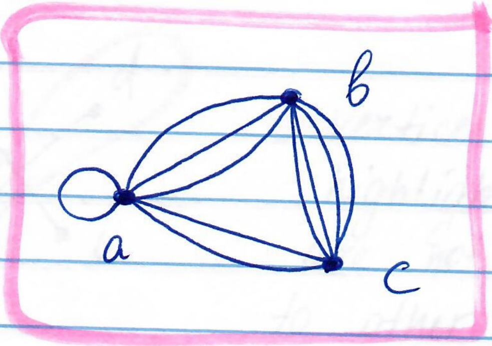
$2 \mid 2, 10$ $3 \mid 3, 15$ $5 \mid 5, 10, 15, 25$ 11



CS135 Sample Final Exam Part 1

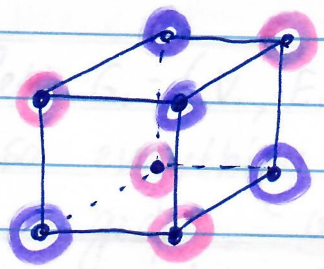
#7

$$\begin{array}{c|c} & \begin{matrix} a & b & c \end{matrix} \\ \hline \begin{matrix} a \\ b \\ c \end{matrix} & \begin{bmatrix} 1 & 3 & 2 \\ 3 & 0 & 4 \\ 2 & 4 & 0 \end{bmatrix} \end{array}$$

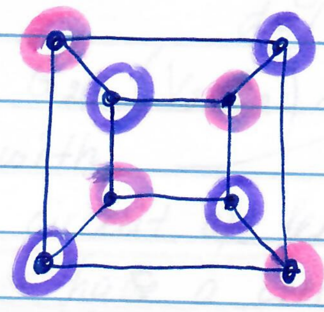


#9

Q_3



or



Yes, it is bipartite. We are able to use 2 colors to color the vertices so that no two adjacent vertices have the same color.

#8.

Adjacency lists:

a	$\to$ b | d | d | d
b	$\to$ a | c | d | e
c	$\to$ b | e | e | d
d	$\to$ a | a | a | b | c | d
e	$\to$ b | c | c | d

or

a	$\to$ (b,1) | (d,3)
b	$\to$ (a,1) | (c,1) | (d,1) | (e,1)
c	$\to$ (b,1) | (e,2) | (d,1)
d	$\to$ (a,3) | (b,1) | (c,1) | (d,1)
e	$\to$ (b,1) | (c,2) | (d,1)

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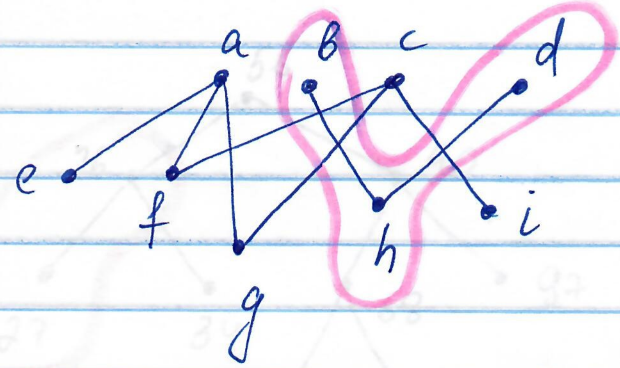
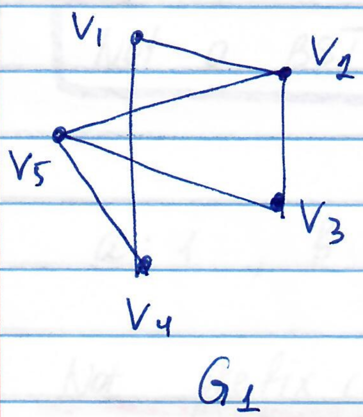
#10

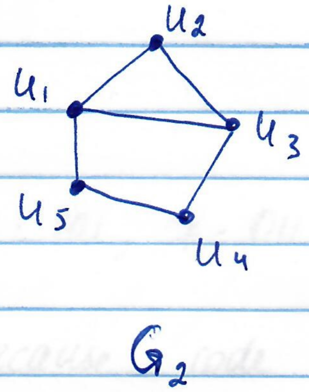


vertices in the highlighted graph (b,d,h) are not connected to other vertices (e,a,f,g,c,i)

$\boxed{\text{Answer: the graph is disconnected.}}$

#11





$|V_1| = |V_2| = 5$

$|E_1| = |E_2| = 6$

$deg(v_5) = deg(v_2) = 3$

$deg(u_1) = deg(u_3) = 3$

$deg(v_1) = deg(v_4) = deg(v_3) = 2$

$deg(u_2) = deg(u_5) = deg(u_4) = 2$

let $G_1 = (V_1, E_1)$ $G_2 = (V_2, E_2)$

so everything is fine with

- graph G_2 has a circuit of length 3 and graph G_1 has a circuit of length 3
- same for the cycle of lengths 5 & 4.

$\boxed{\text{Answer: the graphs are isomorphic}}$

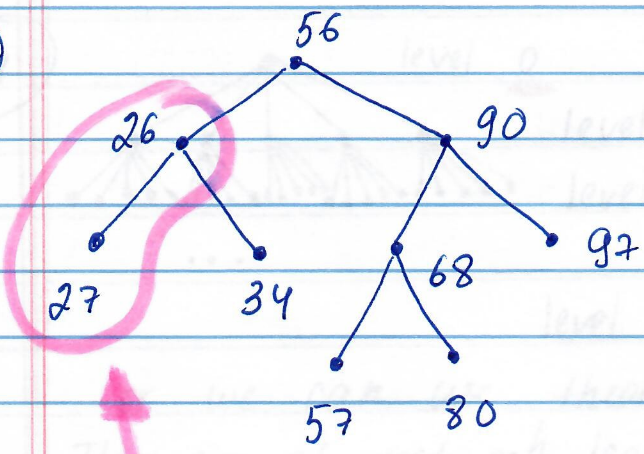
an isomorphism: $V_1 \to u_5$ $V_4 \to u_3$

$V_2 \to u_1$ check edges (remaining):

$V_3 \to u_2$ $\{v_1, v_4\} \to \{u_5, u_4\}$ ✓

$V_5 \to u_3$ $\{v_5, v_2\} \to \{u_1, u_3\}$ ✓

#12



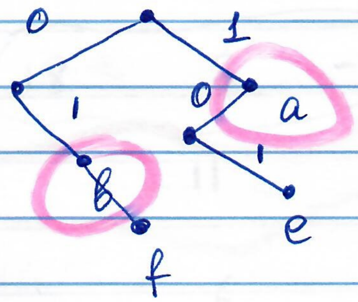
BST property is violated (left child is greater than the parent)

Not a BST.

#13

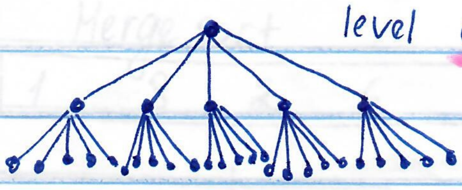
a : 1 , b : 01, e : 101, f : 011 , h : 1000

Not prefix codes, because code for a is a prefix for code for e and h.



CS135 Sample Final Exam Part 1

№14



level 0 $1 = 5^0$ vertices

level 1 $5 = 5^1$ vertices

level 2 $5 \cdot 5 = 5^2$ vertices

… …

level 4 5^4 leaves

or we can use theorem ~~corollary 1~~ (p. 754):
There are at most m^h leaves in an m-ary tree of height h.
- our case is exactly "the most" case, i.e. 5^4 leaves.

$\boxed{\text{Answer: } 5^4 \text{ leaves}}$

№15

6 3 5 2 1 − + * 7 − /

1

6

18

6 18 7 − /

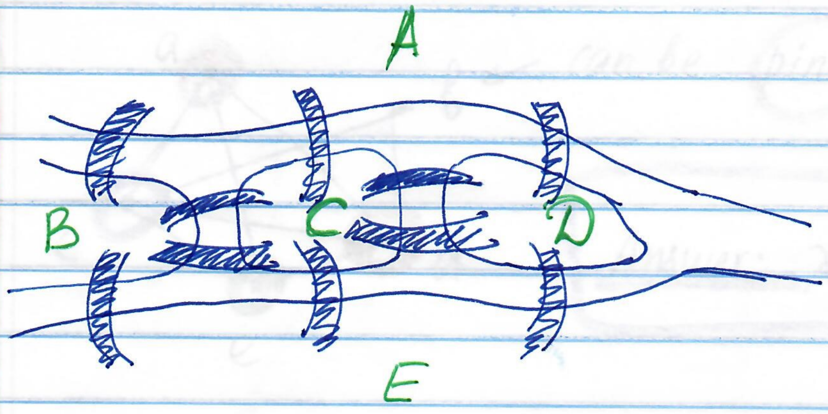
11

$\frac{6}{11}$

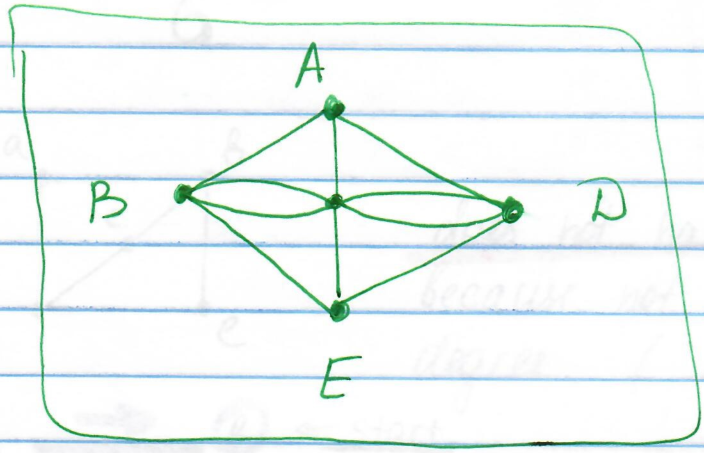
$\boxed{\text{Answer: } \frac{6}{11}}$

CS135 Sample Final Exam Part 2

N°2



edges: bridges
vertices: boroughs (parts)



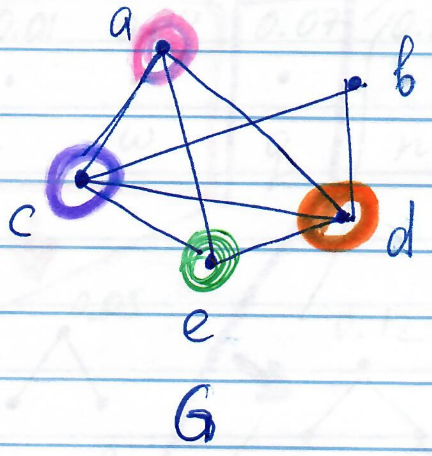
crossing all the bridges exactly once and returning to the starting point is equivalent to finding an Euler circuit in this graph.

The Euler circuit does not exist because deg (A), deg(E) are not even.

Answer: nobody can do it. (it cannot be done)

CS135 **Sample Final Exam** Part 2

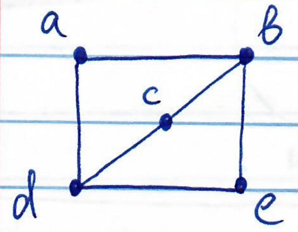
No 3



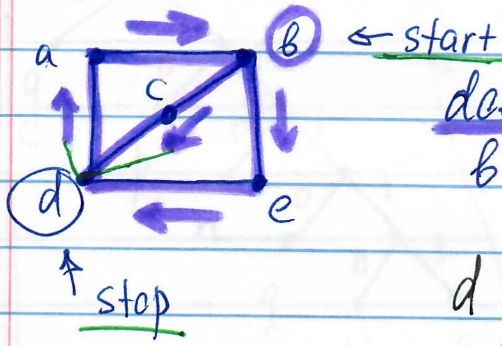
b ← can be pink or green

$$\boxed{\text{Answer: } \chi(G) = 4}$$

No 4



does not have an Euler circuit because not all vertices have even degree (deg(b) = deg(d) = 3)



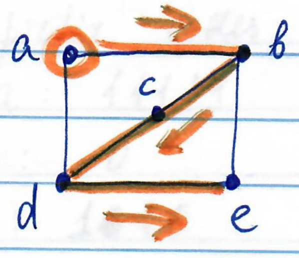
does have an Euler path. (trail)

$b \to c \to d \to a \to b \to e \to d$

or

$d \to a \to b \to e \to d \to c \to b$ ...

does have a Hamilton path.



$a \to b \to c \to d \to e$

or

$a \to b \to e \to d \to c$ ...

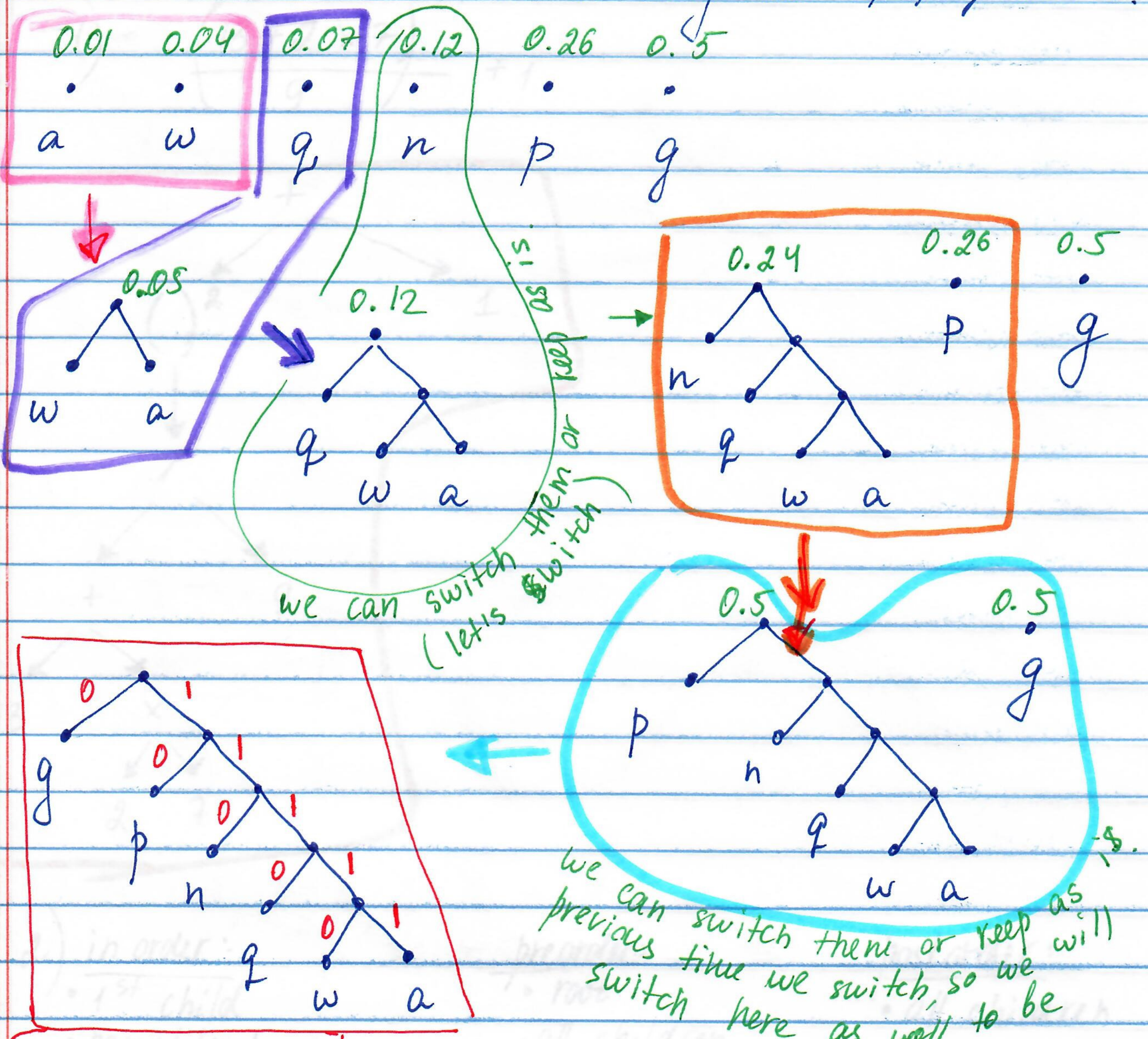
does not have a Hamilton circuit (cycle)

135

Sample Final Exam Part 2.

№5

order the vertices in increasing order of frequencies:



prefix codes:

a : 11111

g : 0

n : 110

p : 10

q : 1110

w : 11110

the average number of bits required to encode a symbol =

$= 5\cdot 0.01 + 1\cdot 0.5 + 3\cdot 0.12 + 2\cdot 0.26 + 4\cdot 0.07 + 5\cdot 0.04 = \underline{1.91}$

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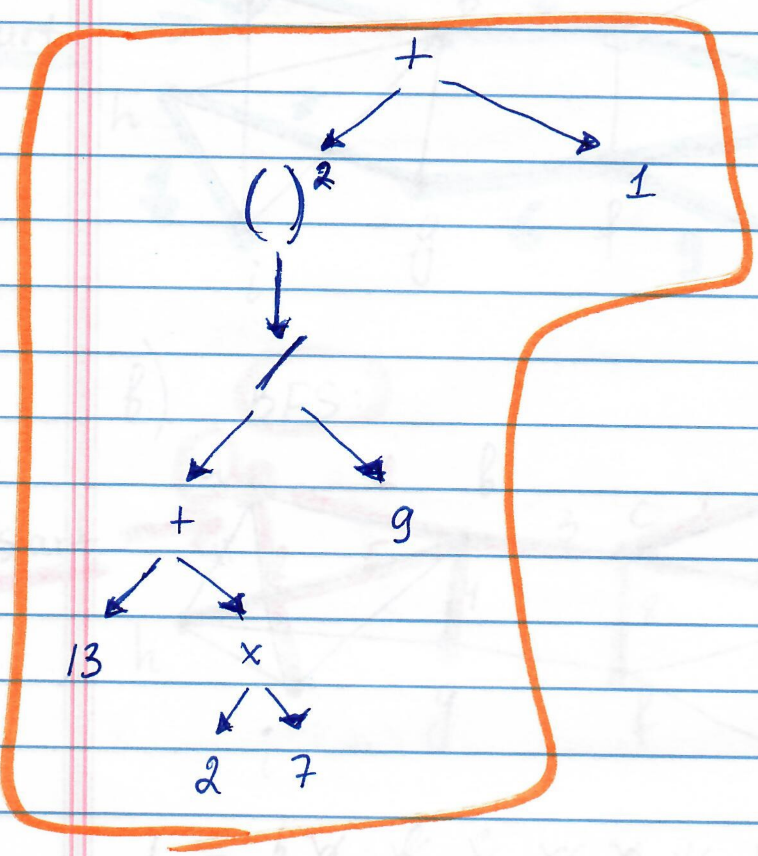
Sample Final Exam Part 2.

№ 6

1) $\left(\dfrac{13+2\times 7}{9}\right)^2 + 1$



2) in order:
- 1st child
- parent / root
- the rest of children

preorder:
- root
- all children (left to right)

postorder:
- all children
- root

a) f, b, g, a, c, d, j, h, k, e, i

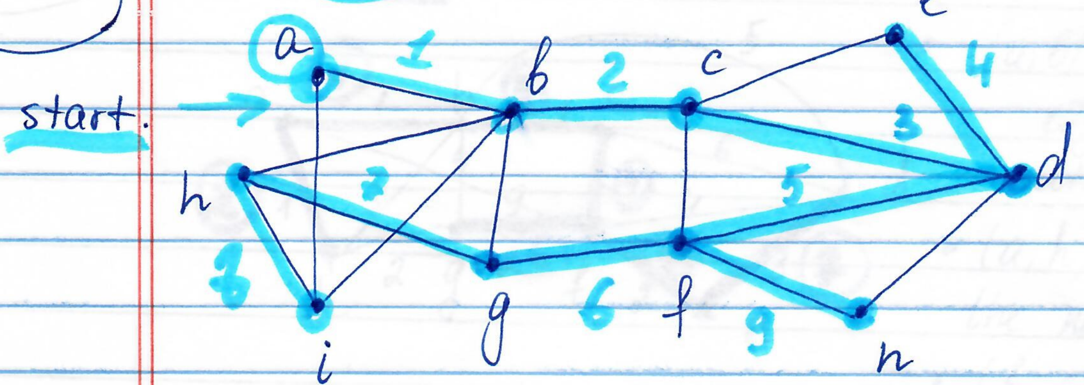
b) a, b, f, g, c, d, e, h, j, k, i

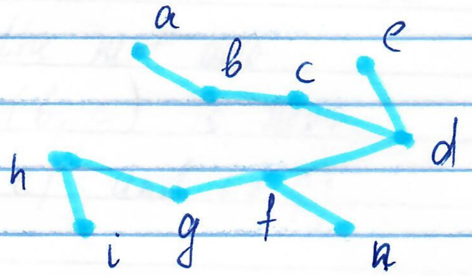
c) f, g, b, c, d, j, k, h, i, e, a

Sample Final Exam Part 2.

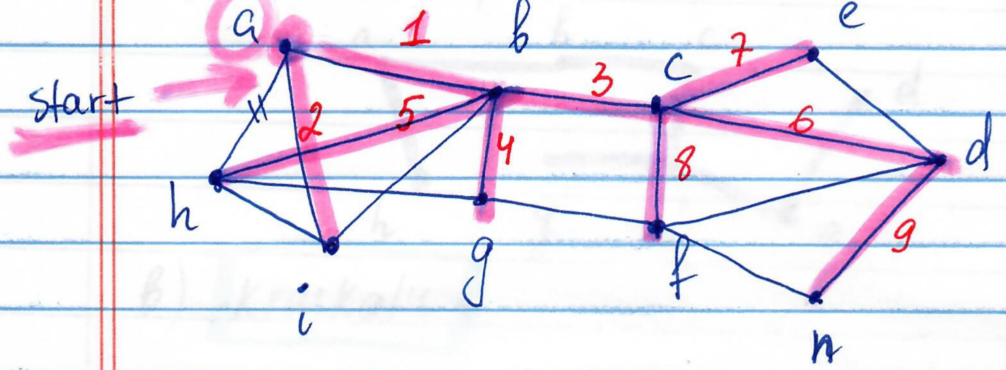
№7

a) DFS:

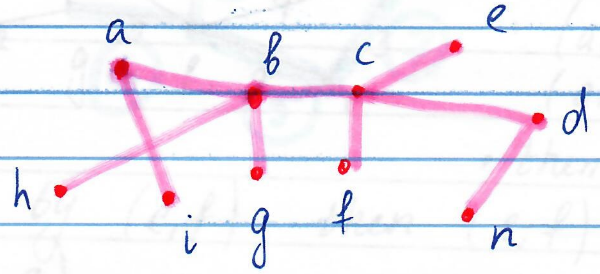




b) BFS:



$L = \{ \cancel{a}_0, \cancel{b}_1, \cancel{i}_2, \cancel{c}_3, \cancel{g}_4, \cancel{h}_5, \cancel{d}_6, \cancel{e}_7, \cancel{f}_8, \cancel{n}_9 \}$

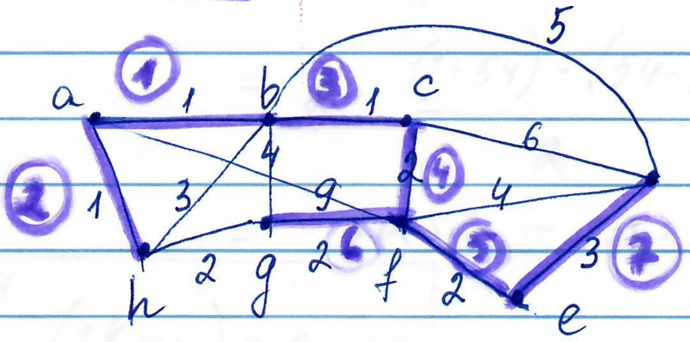


S135

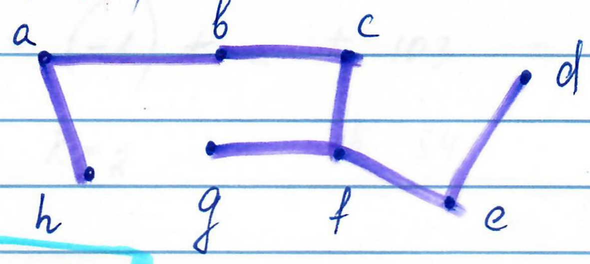
Sample Final Exam

No 8

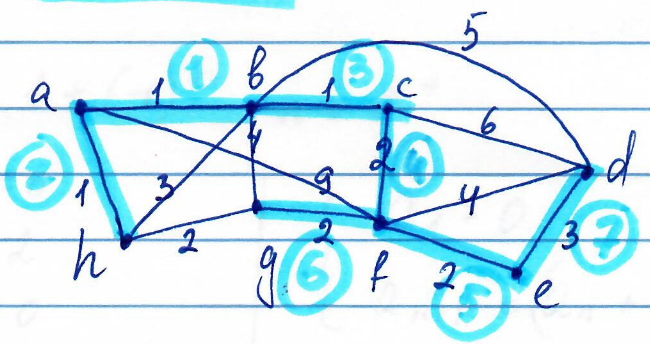
a) $\boxed{\text{Prim's}}$:



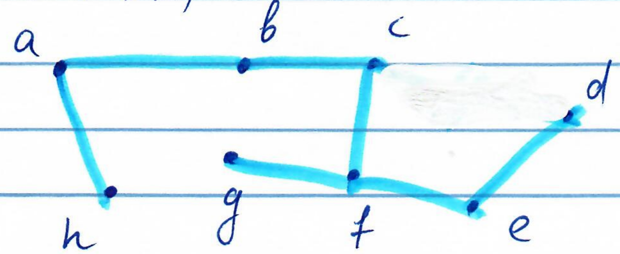
- (a,b) is the first edge to add to the min. spanning tree
- (a,h) let it be the next one.
- (b,c) is next
- then (c,f), followed by (f,e), and (f,g)
- then (e,d)



b) $\boxed{\text{Kruskal's}}$:



- (a,b) is the first edge to be added to the min. spanning tree.
- (a,h) is the next one (alphabetical order)
- then (b,c)
- followed by (c,f), then (e,f)
- then (f,g)
- and finally (d,e)



Note that in this example both algorithms were adding ~~similar~~ same edges in the same order (that is just a coincidence)