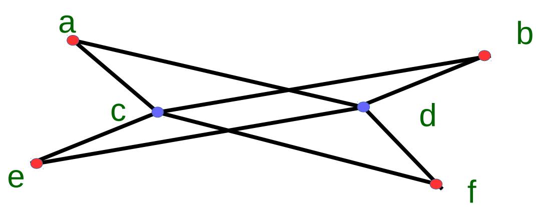
1. Yes, it is possible to color the vertices with two colors, so that no two adjacent vertices have the same color.



2. Graphs are not isomorphic, because the first one has 2 vertices of degree 2, and the second has 3 vertices on degree 2.
Another reason: the second graph has a vertex of degree 4 and the first one does not.

3. The graphs are not isomorphic because graph G has a cycle of length 3 and graph S does not.

4. The graphs are isomorphic. If we move the rows of the matrix M_G around we will get matrix M_H.

5.

a	→	b	b	c				
b	→	a	a	c	c	c	d	
c	→	a	b	b	b	d	f	e
d	→	b	c	f				
e	→	c	f					
f	→	c	d	e				

adjacency list representation

$$\begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ f \end{array} \begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{bmatrix} 0 & 2 & 1 & 0 & 0 & 0 \\ 2 & 0 & 3 & 1 & 0 & 0 \\ 1 & 3 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

matrix representation

6. The first graph is connected. The second one is unconnected – it has three connected components.

7. Yes, it does have cut vertices: u_5 and u_6 and cut edge: $\{u_6,u_5\}$

8. It has a Euler circuit because all vertices have even degree.

An Euler circuit: <a,d,a,d,c,e,c,b,e,d,b,a>

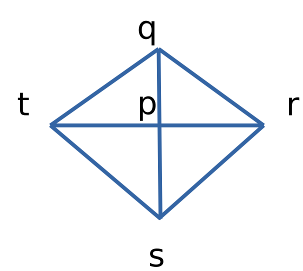
9. It has a Hamilton path and a Hamilton cycle.

a Hamilton path: <p,q,r,s,t>
a Hamilton cycle: <p,q,r,s,t,p>

10. It has a Hamilton path and a Hamilton cycle.

A Hamilton cycle: <a,e,b,c,f,d,a>
a Hamilton path: <a,e,b,c,f,d>

11. The given graph is planar. The given representation is not planar. A planar representation:

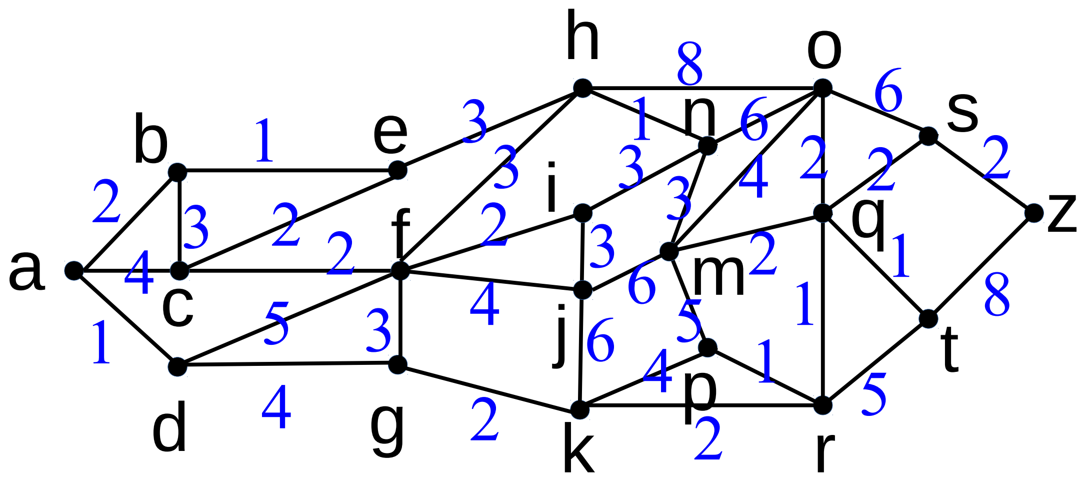


12. $\chi(G) = 4$

Note that subgraph of G with vertices $\{u_1,u_2,u_7,u_6\}$ is a complete graph K_4, i.e. all vertices are connected to each other. Therefore, the minimum number of colors to use is 4.

13. the shortest (in miles) route from Camden city to Newark city:
Camden → Woodbridge → Newark
cost: 80

14. Find the shortest path from a to z in the given graph G., using Dijkstra's algorithm for weighted graphs.



G

15. K_5: 5 edges, each edge is connected to 4 other vertices: 5*4,
but this way we count each vertex twice, therefore we need to divide by 2:

5 × 4 ÷ 2 = 10 edges K_5 is a 4-regular graph

16. Such a graph is not possible.

3-regular graph with 5 vertices should have 5 × 3 ÷ 2 edges.
15 is not divisible by 2. Therefore such a graph is not possible.

or

use the handshaking theorem: $2m = \sum_{v \in V} deg(v) = 2|E|$

$\sum_{v \in V} deg(v) = 5 * 3 = 15$ and it should be even (= 2m), but it is not!

17. There is no longest possible walk in a graph with n vertices, because we can make it as long as we wish.

18. A cycle of length n.

19. the number of edges in K_6 is 6 × 5 ÷ 2 = 15

By the theorem: **[Theorem]** *Number of edges in a planar graph*
Consider a connected planar simple graph G=(V,E), where |V| = n, |E| = m with n ≥ 3, then
m ≤ 3n − 6

15 must be ≤ 3× 6 − 6 = 12 contradiction
Therefore, K_6 is not planar.