BRONX COMMUNITY COLLEGE

of The City University of New York

DEPARTMENT OF MATHEMATICS and COMPUTER SCIENCE

MATH 13

Final Exam Review problems

1. Draw the given vectors, find their x- and y-components (to the nearest hundrenth) and find their sum. Angles are given in standard position.

   $A = 7.5\text{cm}$, $\theta = 240°$, $B = 2.3\text{cm}$, $\theta = 30°$

2. With the given set of components, find R(magnitude) and θ (direction, angle): $R_x = 7.627$, $R_y = -6.353$

3. Simplify and write $7 + \sqrt{-\frac{50}{144}}$ as complex number in rectangular form.

4. Perform the indicated operations and simplify each complex number to its rectangular form.
   a. $2j - \sqrt{-150}$ **b.** $(\sqrt{-3})^2 + j^6$

5. Perform the indicated operations. Express all answers in the rectangular form $(a + bj)$.
   a. $(-4 - j) + (-7 - 4j)$ **b.** $(-4 - j) - (-7 - 4j)$
   c. $(-4 - j) \cdot (-7 - 4j)$ **d.** $\frac{-4-j}{-7-4j}$

6. Represent the complex number $-6 + 4j$ graphically and give its polar form.

7. Represent the complex number $1 - 5j$ graphically and give its exponential form.

8. Express the complex number $3(\cos 232° + j \sin 232°)$ in rectangular form.

9. Express the complex number $2.5e^{3.8j}$ in polar and rectangular forms.

10. Perform the indicated operations and give the answer in polar form.
    a. $(15.9\angle 142.6°)^4$ **b.** $(0.4\angle 320°) \cdot (5.5\angle - 150°)$ **c.** $\frac{0.4\angle 320°}{5.5\angle -150°}$

11. $f(x) = \begin{cases} \sqrt{x}+2, & \text{if } x \geq 0 \\ 2x-1, & \text{if } x < 0 \end{cases}$ Find $f(7)$ and $f(-2)$

12. $h(x) = 5 + 2x$. Find $h(5) - h(2)$.

13. Find the domain and the range of the given functions: **(a)** $f(x) = \sqrt{x - 10}$ **(b)** $g(x) = \frac{x+2}{x-5}$

14. Graph the given functions.
    (a) $f(x) = 2x^2 + 5$ **(b)** $f(x) = \frac{2}{x}$
    (c) $g(x) = \sqrt{2x + 1}$ **(d)** $k(x) = 2x^3$

15. Express the given equations in logarithmic form: **(a)** $3^{-3} = \frac{1}{27}$ **(b)** $4^{\frac{5}{2}} = 32$

16. Express the given equations in exponential form: **(a)** $\log_4 1024 = 5$ **(b)** $\log_{\frac{1}{7}} 343 = -3$

17. Determine the value of the unknown: **(a)** $\log_2 4 = x$ **(b)** $\log_{\frac{1}{3}} y = 2$

18. Express the logarithm $\log_7 \frac{y^2}{49}$ as a sum, difference, or a multiple of logarithms.

19. Express each of the logarithms as a single quantity

    (a) $-\log_2 x + \frac{1}{2}\log_2 81$ **(b)** $2\log_4 5 + 4\log_4 a^2 + \log_4 \frac{1}{7}$

20. Find: **(a)** $\log_{23} 132$ **(b)** $\log_{\frac{1}{2}} 7$

    (hint: use commong or natural logarithms)

21. Determine the amplitude, period and displacement for each function:

    (a) $y = 3\sin(x - \frac{\pi}{4})$ **(b)** $y = -4\cos(2x + \frac{\pi}{3})$ **(c)** $y = 210\sin(3\pi x)$

22. Sketch the graph of each function:

    (a) $y = \sin(3x)$ **(b)** $y = -3\cos(2x)$ **(c)** $y = 2\tan(x + \frac{\pi}{2})$
    (d) $y = 40\cos(3\pi x + 2)$ **(e)** $y = -2\csc x$

23. Prove the given identities:

    (a) $\sin x \sec x = \tan x$ (b) $\csc^2 x(1 - cos^2 x) = 1$ (c) $\cot\theta\sec^2\theta - \cot\theta = \tan\theta$
    (d) $\frac{1+\cos x}{\sin x} = \frac{\sin x}{1-\cos x}$ (e) $\cos(-x) = \cos x$ (f) $\sin(\frac{\pi}{2} + x) = \cos x$
    (g) $\cos^2\frac{x}{2}(1 + (\frac{\sin x}{1+\cos x})^2) = 1$ (h) $\frac{\sin 4\theta}{\sin 2\theta} = 2\cos 2\theta$ (i) $1 - \cos 2\theta = \frac{2}{1+\cot^2\theta}$

24. Find $\sin 150°$ by using $150° = 60° + 90°$

25. Find $\sin 4x$ if $\sin x = 0.6$ (first quadrant)

26. Find $\sin\frac{\theta}{2}$ if $\cos\theta = \frac{13}{14}$ $(0° < \theta < 90°)$

27. Find $\cos\frac{x}{2}$ if $\tan x = -0.2917$ $(90° < x < 180°)$

28. Solve trigonometric equations for values of x for $0 \le x < 2\pi$:

    (a) $4\sin^2 x - 1 = 0$ **(b)** $2\cos^2 x - \cos x = 0$

29. Solve the given equations for x in terms of y:

    (a) $y - 6 = \tan^{-1}\frac{x}{4}$ **(b)** $y = 2\sin^{-1}\frac{x}{6}$

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ANSWERS:

1. $R \approx 5.63$, $\theta = 251.79°$ **2.** $R = 9.93$, $\theta = -39.79°$ **3.** $7 + j\frac{5\sqrt{2}}{12}$ **4. a.**$j(2 - 5\sqrt{6})$ **b.**-4
5. a.$-11 - 5j$ **b.**$3 + 3j$ **c.**$24 + 23j$ **d.**$\frac{32-9j}{65}$ **6.** $2\sqrt{13}\angle 146.31°$ **7.** $\sqrt{26}e^{-1.37j}$
8. $-1.85 - 2.36j$ **9.** $2.5\angle 217.72° = -1.98 - 1.53j$ **10. a.** $63912.9\angle 210.4°$ **b.**$2.2\angle 170°$ **c.** $0.07\angle 110°$ **11.** $f(7) \approx 4.65$, $f(-2) = -5$ **12.** 6 **13.** see solutions **14.** see solutions
15. a. $\log_3 \frac{1}{27} = -3$ **b.** $\log_4 32 = \frac{5}{2}$ **16. a.** $4^5 = 1024$, **b.** $(\frac{1}{7})^{-3} = 343$ **17. a.** $x = 2$ **b.** $y = \frac{1}{9}$ **18.** $2\log_7 y - 2$ **19. a.**$\log_2(\frac{9}{x})$ **b.** $\log_4(\frac{25\cdot a^8}{7})$ **20. a.** 1.56 **b.** -2.81 **21.** see solutions **22.** see solutions **23.** see solutions for proofs **24.** $\frac{1}{2}$ **25.** 0.5376
26. $\frac{\sqrt{7}}{14}$ **28. a.** $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ **b.** $x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{3}, \frac{5\pi}{3}$ **29. a.** $x = 4\cdot\tan(y - 6)$ **b.** $x = 6\cdot\sin\frac{y}{2}$ **30.** -1485 **31.** $x = -\frac{6}{19}$, $y = \frac{36}{19} = 1\frac{17}{19}$ **32.** -115

SOLUTIONS:

1. $A_x = -3.75$, $B_x = 1.99$ thus $R_x = A_x + B_x = -1.79$;
$A_y = -6.5, B_y = 1.15$, thus $R_y = A_y + B_y = -5.35$
$R = \sqrt{R_x^2 + R_y^2} \approx 5.63$, $\theta_{ref} = \tan^{-1}(\frac{R_y}{R_x}) = 71.79°$, $\theta_{standard} = 251.79$

2. $R = \sqrt{(7.627)^2 + (-6.353)^2} = 9.93$; $\quad \theta = \tan^{-1} \frac{-6.353}{7.627} = -39.79°$ or $\theta_{stand.} = 320.21°$

3. $7 + \sqrt{-\frac{50}{144}} = 7 + j\sqrt{\frac{50}{144}} = 7 + j\frac{5\sqrt{2}}{12}$

4. **a.** $2j - \sqrt{-150} = 2j - \sqrt{25 \cdot 6 \cdot (-1)} = 2j - 5j\sqrt{6} = j(2 - 5\sqrt{6})$
b. $(\sqrt{-3})^2 + j^6 = (\sqrt{3}j) + (j^2)^3 = -3 + (-1)^3 = -3 - 1 = -4$

5. **a.** $(-4 - j) + (-7 - 4j) = -4 - j - 7 - 4j = -11 - 5j$
b. $(-4 - j) - (-7 - 4j) = -4 - j + 7 + 4j = 3 + 3j$
c. $(-4 - j) \cdot (-7 - 4j) = 28 + 16j + 7j + 4j^2 = 24 + 23j$
d. $\frac{-4-j}{-7-4j} = \frac{(-4-j)(-7+4j)}{(-7-4j)(-7+4j)} = \frac{28-16j+7j-4j^2}{49-16j^2} = \frac{32-9j}{65}$

6. $r = \sqrt{(-6)^2 + (4)^2} = \sqrt{52} = 2\sqrt{13}$, $\quad \theta_{ref} = \tan^{-1} \frac{4}{-6} = -33.69°$,
$\theta = 180 - 33.69 = 146.31°$. Thus $-6 + 4j = 2\sqrt{13}\angle 146.31°$

7. $r = \sqrt{1 + 25} = \sqrt{26}$, $\quad \theta_{ref} = \tan^{-1} \frac{-5}{1} = -78.69°$ or $281.31°$;
radians$=\frac{281.31° \cdot \pi}{180°} \approx 4.91$. $\quad$ Thus $1 - 5j = \sqrt{26}\angle 281.31° = \sqrt{26}\, e^{4.91j}$

8. $a = 3\cos(232°) = -1.85$, $b = 3\sin(232°) = -2.36$. Thus $3\angle 232° = -1.85 - 2.36j$

9. $degrees = \frac{3.8 \cdot 180°}{\pi} = 217.72 \quad a = 2.5\cos 217.72° = -1.98, \quad b = 2.5\sin 217.72° = -1.53.$
Thus $2.5e^{3.8j} = 2.5\angle 217.72° = -1.98 - 1.53j$

10. **a.** $(15.9\angle 142.6°)^4 = (15.9)^4\angle(4 \cdot 142.6°) = 63912.9\angle 570.4° = 63912.9\angle 210.4°$
b. $(0.4\angle 320°) \cdot (5.5\angle - 150°) = (0.4 \cdot 5.5)\angle(320° + (-150°)) = 2.2\angle 170°$
c. $\frac{0.4\angle 320°}{5.5\angle -150°} = \frac{0.4}{5.5}\angle(320° - (-150°)) = 0.07\angle 470° = 0.07\angle 110°$

11. $f(7) = \sqrt{7} + 2 \approx 4.65$, $f(-2) = 2 \cdot (-2) - 1 = -5$

12. $h(5) = 5 + 10 = 15$, $\quad h(2) = 5 + 4 = 9$, $\quad$ thus $h(5) - h(2) = 15 - 9 = 6$

13. **a.** domain: $x \geq 10$, range: $f(x) \geq 0$, **b.** domain: $x \neq 5$, range: all real numbers.

14. see pictures at the end of the handout

15. **a.** $\log_3 \frac{1}{27} = -3 \quad$ **b.** $\log_4 32 = \frac{5}{2}$

16. **a.** $4^5 = 1024 \quad$ **b.** $(\frac{1}{7})^{-3} = 343$

17. **a.** exponential form: $2^x = 4$, thus $x = 2 \quad$ **b.** exponential form: $(\frac{1}{3})^2 = y$, thus $y = \frac{1}{9}$

18. $\log_7 \frac{y^2}{49} = \log_7(y^2) - \log_7 49 = 2\log_7 y - 2$

19. **a.** $-\log_2 x + \frac{1}{2}\log_2 81 = \log_2 x^{-1} + \log_2 81^{\frac{1}{2}} = \log_2(\frac{1}{x}) + \log_2 9 = \log_2(\frac{9}{x})$
b. $2\log_4 5 + 4\log_4 a^2 + \log_4 \frac{1}{7} = \log_4 25 + \log_4 a^8 + \log_4(\frac{1}{7}) = \log_4(\frac{25 \cdot a^8}{7})$

20. **a.** $\log_{23} 132 = \frac{\log 132}{\log 23} \approx 1.56 \quad$ **b.** $\log_{\frac{1}{2}} 7 = \frac{\ln 7}{\ln \frac{1}{2}} \approx -2.81$

21. **a.** amplitude=3, period=2π, displacement=$\frac{\pi}{4}$
    b. amplitude=4, period=$\frac{2\pi}{2=\pi}$, displacement=$-\frac{\frac{\pi}{3}}{2} = -\frac{\pi}{6}$
    c. amplitude=210, period=$\frac{2\pi}{3\pi} = \frac{2}{3}$, displacement=0

22. see pictures at the end of the handout

23. (a) $\sin x \sec x = \tan x$; $\quad \sin x \sec x = \sin x \cdot \frac{1}{\cos x} = \frac{\sin x}{\cos x} = \tan x$

    (b) $\csc^2 x(1 - cos^2 x) = 1$; $\quad \csc^2 x(1 - cos^2 x) = \frac{1}{\sin^2 x} \cdot \sin^2 x = \frac{\sin^2 x}{\sin^2 x} = 1$

    (c) $\cot\theta \sec^2\theta - \cot\theta = \tan\theta$; $\quad \cot\theta\sec^2\theta - \cot\theta = \frac{1}{\tan\theta}(\frac{1}{\cos^2\theta} - 1) = \frac{1}{\tan\theta}(\frac{1-\cos^2\theta}{\cos^2\theta}) = \frac{1}{\tan\theta}\frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\tan\theta}\tan^2\theta = \tan\theta$

    (d) $\dfrac{1+\cos x}{\sin x} = \dfrac{\sin x}{1-\cos x}$; $\quad \dfrac{1+\cos x}{\sin x} =^? \dfrac{\sin x}{1-\cos x}$, $\quad (1+\cos x)(1-\cos x) =^? \sin^2 x$, $1 - \cos^2 x = \sin^2 x$. Thus our assumption was true and the identity is true.

    (e) $\cos(-x) = \cos x$; $\quad \cos(-x) = \cos(0° - x) = \cos 0° \cos x - \sin 0° \sin x = \cos x$. $(\cos 0° = 1, \sin 0° = 0)$

    (f) $\sin(\frac{\pi}{2} + x) = \cos x$; $\quad \sin(\frac{\pi}{2} + x) = \sin\frac{\pi}{2}\cos x - \cos\frac{\pi}{2}\sin x = \cos x$.
    $(\sin\frac{\pi}{2} = 1, \cos\frac{\pi}{2} = 0)$

    (g) $\cos^2\frac{x}{2}(1 + (\frac{\sin x}{1+\cos x})^2) = 1$
    $\cos^2\frac{x}{2}(1+(\frac{\sin x}{1+\cos x})^2) = \frac{1+\cos x}{2}(1+(\frac{\sin x}{1+\cos x})^2) = \frac{1+\cos x}{2}\cdot(\frac{(1+\cos x)^2+\sin^2 x}{(1+\cos x)^2}) = \frac{1+\cos x}{2}\cdot \frac{1+\cos^2 x+\sin^2 x+2\cos x}{(1+\cos x)^2} = \frac{1}{2}\cdot\frac{1+1+2\cos x}{1+\cos x} = \frac{1}{2}\cdot\frac{2(1+\cos x)}{1+\cos x} = 1$

    (h) $\dfrac{\sin 4\theta}{\sin 2\theta} = 2\cos 2\theta$; $\quad \dfrac{\sin 4\theta}{\sin 2\theta} = \dfrac{2\sin 2\theta\cos 2\theta}{\sin 2\theta} = 2\cos 2\theta$

    (i) $1 - \cos 2\theta = \frac{2}{1+\cos^2\theta}$; $\quad \frac{2}{1+\cot^2\theta} = \frac{2}{\csc^2\theta} = \dfrac{2}{\frac{1}{\sin^2\theta}} = 2\sin^2\theta = 1 - \cos 2\theta$

24. $\sin 150° = \sin(60° + 90°) = \sin 60°\cos 90° + \sin 90°\cos 60° = \frac{\sqrt{3}}{2}\cdot 0 + 1\cdot\frac{1}{2} = \frac{1}{2}$

25. $\sin 4x = 2\sin 2x\cos 2x = 2\cdot 2\sin x\cos x\cdot(1 - 2\sin^2 x) = 2\cdot 2\sin x \pm \sqrt{1-\sin^2 x}\cdot(1 - 2\sin^2 x) = 2\cdot 2\cdot 0.6\cdot\sqrt{1-0.6^2}\cdot(1 - 2\cdot 0.6^2) \approx 0.5376$

26. $\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}} = \sqrt{\frac{1-\frac{13}{14}}{2}} = \sqrt{\frac{1}{14\cdot 2}} = \frac{1}{2\sqrt{7}} = \frac{\sqrt{7}}{14}$

27. (a) $4\sin^2 x - 1 = 0$
    $\sin^2 x = \frac{1}{4}$, i.e. $\sin x = \pm\sqrt{\frac{1}{4}}$, i.e. $\sin x = \pm\frac{1}{2}$, $\quad$ thus $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

    (b) $2\cos^2 x - \cos x = 0$
    $2\cos^2 x - \cos x = \cos x(2\cos x - 1) = 0$, i.e.
    $\cos x = 0$ and thus $x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\quad$ or $\quad \cos x = \frac{1}{2}$, $\quad$ thus $x = \frac{\pi}{3}, \frac{5\pi}{3}$

28. (a) $y - 6 = \tan^{-1}\frac{x}{4}$, take tan of both sides: $\tan(y-6) = \frac{x}{4}$, i.e. $x = 4\cdot\tan(y-6)$

    (b) $y = 2\sin^{-1}\frac{x}{6}$, divide both sides by $2: \frac{y}{2} = \sin^{-1}\frac{x}{6}$,
    take sin of both sides: $\sin\frac{y}{2} = \frac{x}{6}$, i.e. $x = 6\cdot\sin\frac{y}{2}$

29. $-18 \quad -33$
    $-21 \quad 44 = (-18)\cdot 44 - (-21)\cdot(-33) = -1485$

30. re-arrange terms: we need to evaluate the following determinants:

$7x - 2y = -6$ $\quad$ $\begin{matrix} 7 & -2 \\ 4 & 7 \end{matrix}$ $\quad$ $\begin{matrix} -6 & -2 \\ 12 & 7 \end{matrix}$ $\quad$ $\begin{matrix} 7 & -6 \\ 4 & 12 \end{matrix}$

$4x + 7y = 12$

Let's evaluate the denominator's determinant: $7 \cdot 7 - (-2)4 = 49 + 8 = 57$

$x = \frac{(-6)7-(-2)12}{57} = \frac{-18}{57} = -\frac{6}{19}$; $\quad y = \frac{7\cdot12-4(-6)}{57} = \frac{108}{57} = \frac{36}{19} = 1\frac{17}{19}$