BCC/CUNY

MATH 13

Test 3 Sample with Answers/Solutions

1. Determine the amplitude, period and displacement for each function:

   (a) $y = 3\sin(x - \frac{\pi}{4})$
   Answer: amplitude=3, period=2π, displacement=$\frac{\pi}{4}$

   (b) $y = -4\cos(2x + \frac{\pi}{3})$
   Answer: amplitude=4, period=π, displacement=$-\frac{\pi}{6}$

   (c) $y = 210\sin(3\pi x)$
   Answer: amplitude=210, period=$\frac{2}{3}$, displacement=0

2. Sketch the graph of each function:

   (a) $y = \sin(3x)$

   (b) $y = -3\cos(2x)$

   (c) $y = 2\tan(x + \frac{\pi}{2})$

   (d) $y = 40\cos(3\pi x + 2)$

   (e) $y = -2\csc x$
   Answer: see the graphs at the end

3. Prove the given identities:

   (a) $\sin x \sec x = \tan x$
   Solution: $\sin x \sec x = \sin x \cdot \frac{1}{\cos x} = \frac{\sin x}{\cos x} = \tan x$

   (b) $\csc^2 x(1 - cos^2) = 1$
   Solution: $\csc^2 x(1 - cos^2) = \frac{1}{\sin^2 x} \cdot \sin^2 x = 1$

   (c) $\cot\theta \sec^2\theta - \cot\theta = \tan\theta$
   Solution: $\cot\theta\sec^2\theta - \cot\theta = \cot\theta(\sec^2\theta - 1) = \cot\theta(\frac{1}{\cos^2\theta} - 1) = \cot\theta\frac{1-\cos^2\theta}{\cos^2\theta} = \frac{\cos\theta}{\sin\theta}\frac{\sin^2\theta}{\cos^2\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta$

   (d) $\frac{1+\cos x}{\sin x} = \frac{\sin x}{1-\cos x}$
   Solution: use cross-product: $(1 + \cos x)(1 - \cos x) = \sin x \cdot \sin x$ or $1 - \cos^2 x = \sin^2 x$ or $\sin^2 x = \sin^2 x$. Thus the identity is true.

   (e) $\cos(-x) = \cos x$
   Solution: $\cos(-x) = \cos(0 - x) = \cos 0 \cos x - \sin 0 \sin x = \cos x$

   (f) $\sin(\frac{\pi}{2} + x) = \cos x$
   Solution: $\sin(\frac{\pi}{2} + x) = \sin\frac{\pi}{2}\cos x + \sin x \cos\frac{\pi}{2} = \cos x$

   (g) $\cos^2\frac{x}{2}(1 + (\frac{\sin x}{1+\cos x})^2) = 1$
   Solution: $\cos^2\frac{x}{2}(1 + (\frac{\sin x}{1+\cos x})^2) = \left(\pm\sqrt{\frac{1+\cos x}{2}}\right)^2 \cdot (\frac{(1+\cos x)^2+\sin^2 x}{(1+\cos x)^2}) = \frac{1+\cos x}{2} \cdot \frac{1+\cos^2 x+2\cos x+\sin^2 x}{(1+\cos x)^2} = \frac{1+\cos x}{2} \cdot \frac{2+2\cos x}{(1+\cos x)^2} = \frac{1+\cos x}{2} \cdot \frac{2(1+\cos x)}{(1+\cos x)^2} = 1$

   (h) $\frac{\sin 4\theta}{\sin 2\theta} = 2\cos 2\theta$
   Solution: $\frac{\sin 4\theta}{\sin 2\theta} = \frac{2\sin 2\theta\cdot\cos 2\theta}{\sin 2\theta} = 2\cos 2\theta$

(i) $1-\cos 2\theta = \frac{2}{1+\cot^2\theta}$

Solution: $\frac{2}{1+\cot^2\theta} = \frac{2}{1+\frac{\cos^2 x}{\sin^2 x}} = \frac{2}{\frac{\sin^2 x+\cos^2 x}{\sin^2 x}} = \frac{2\sin^2 x}{\sin^2 x+\cos^2 x} = 2\sin^2 x$

4. Reduce each of the given expressions to a single term.

   (a) $\cos(x+y)\cos y + \sin(x+y)\sin y$
   Answer: $\cos x$

   (b) $2\cos^2(\frac{1}{2}x) - 1$
   Answer: $\cos 2x$

5. Find $\sin 150°$ by using $150° = 60° + 90°$

   Solution: $\sin 150° = \sin(60° + 90°) = \sin 60° \cos 90° + \sin 90° \cos 60° = 0 + \frac{1}{2} = \frac{1}{2}$

6. Find $\sin 4x$ if $\sin x = 0.6$ (first quadrant)

   Solution: $\sin 4x = 2\sin 2x \cdot \cos 2x = 2\cdot 2\sin x \cdot \cos x \cdot (1-2\sin^2 x) = 4\cdot 0.6 \cdot \pm\sqrt{1-\sin^2 x} \cdot (1-2\cdot 0.6^2) = 4\cdot 0.6 \cdot \sqrt{1-0.6^2} \cdot (1-2\cdot 0.6^2) = 0.5376$

7. Find $\sin\frac{\theta}{2}$ if $\cos\theta = \frac{13}{14}$ $(0° < \theta < 90°)$

   Solution: $\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}} = \sqrt{\frac{1-\frac{13}{14}}{2}} = \sqrt{\frac{1}{14\cdot 2}} = \frac{\sqrt{7}}{14} \approx 0.2$

8. Find $\cos\frac{x}{2}$ if $\tan x = -0.2917$ $(90° < x < 180°)$

   Solution: $1+\tan^2 x = \sec^2 x$, thus $1+\tan^2 x = \frac{1}{\cos^2 x}$.
   Therefore $\cos^2 x = \frac{1}{1+\tan^2 x}$, and $\cos x = \pm\sqrt{\frac{1}{1+\tan^2 x}} = -\sqrt{\frac{1}{1+(-0.2917)^2}} \approx -0.96$
   Then, $\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}} = \sqrt{\frac{1+(-0.96)}{2}} \approx 0.14$

9. Solve trigonometric equations for values of x for $0 \le x < 2\pi$:

   (a) $4\sin^2 x - 1 = 0$
   Solution: $(2\sin x - 1)(2\sin x + 1) = 0$, i.e., $(2\sin x - 1) = 0$ or $(2\sin x + 1) = 0$, i.e., $\sin x = \frac{1}{2}$ or $\sin x = -\frac{1}{2}$, i.e. $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$.

   (b) $2\cos^2 x - \cos x = 0$
   Solution: $\cos x(2\cos x - 1) = 0$, i.e., $\cos x = 0$ or $2\cos x - 1 = 0$, i.e., $x = \frac{\pi}{2}, \frac{3\pi}{2}$ or $\cos x = \frac{1}{2}$, i.e., $x = \frac{\pi}{2}, \frac{\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}$.

10. Solve the given equations for x:

   (a) $y - 6 = \tan^{-1}\frac{x}{4}$
   Solution: $\tan(y-6) = \frac{x}{4}$, i.e. $x = 4\cdot\tan(y-6)$

   (b) $y = 2\sin^{-1}\frac{x}{6}$
   Solution: $\frac{y}{2} = \sin^{-1}\frac{x}{6}$, i.e., $\sin\frac{y}{2} = \frac{x}{6}$, i.e., $x = 6\cdot\sin\frac{y}{2}$