Some Probability Rules – Compound Events

Example 1:
You throw two fair dice. What is the probability of getting a 5 on each die?

1. What are the events we are looking for (favorable events)?

2. Are they independent?

3. What is the *sample space*?

4. What is the probability of getting a 5 on each die?

Some Probability Rules – Compound Events

Example 1:
You throw two fair dice. What is the probability of getting a 5 on each die?

1. What are the events we are looking for (favorable events)?
   Getting 5 on the first die, getting 5 on the second one
2. Are they independent?
   Yes, they are (two different dice)
3. What is the *sample space*?
   The *sample space* for throwing **one** die:
      1, 2, 3, 4, 5, 6 (dots)
   The size of a sample space (|sample space|) is 6.
   - that's all we will need.

   Just to let you know that the sample space of throwing **two dice** is:
   (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), ..., (6,5), (6,6)
   and |sample space|=6 × 6 = 36.

4. What is the probability of getting a 5 on each die?

P(5 on 1st die and 5 on 2nd die)=P(5 on 1st die) × P(5 on 2nd die) = $\left(\frac{1}{6}\right)\times\left(\frac{1}{6}\right)=\frac{1}{36}$

Answer: P(5 on 1st die and 5 on 2nd die)= 1 / 36

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Example 2:
You have a well-shuffled deck of cards (52 cards, four suits: diamonds (♦), spades (♠), hearts (♥) and clubs (♣)). What is the probability of drawing two aces from this deck if the first card is not placed back into a deck before the second one is drawn (without replacement)?

1. What are the events we are looking for (favorable events)?

2. Are they independent?

3. What is the *sample space*?

4. What is the probability of drawing two aces?

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Example 2:
You have a well-shuffled deck of cards (52 cards, four suits: diamonds (♦), spades (♠), hearts (♥) and clubs (♣)). What is the probability of drawing two aces from this deck if the first card is not placed back into a deck before the second one is drawn (without replacement)?

1. What are the events we are looking for (favorable events)?
   draw two aces from a deck of cards (without replacement)
2. Are they independent?
   no, they are not.
   When we draw the first card - we don't return it back, right before the second draw there is one card less in the deck.
   Moreover, if the first card was ace, then there are only three aces left in the deck.
3. What is the *sample space*?
   The *sample space* for the **first draw**: 52 card faces:
   The sample space for the **second draw**: 51 card faces (that are left)

   Just to let you know that the sample space of drawing **two cards** is:
   (2♥,2♦),..., (King♠,Queen♥),...
   and |sample space |=52 × 51 = 2652.

4. What is the probability of drawing two aces?

P(Ace,Ace)=
P(Ace) × P(Ace, given that we drew Ace on the first draw) = $\left(\frac{4}{52}\right)\times\left(\frac{3}{51}\right)=\frac{12}{2652}\approx 0.0045$

or 0.45% chance to draw two aces

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Example 3:
You have a well-shuffled deck of cards (52 cards, four suits: diamonds (♦), spades (♠), hearts (♥) and clubs (♣)). Drawing one card. What is the probability of drawing either a jack or a king?

1. What are the events we are looking for?

2. Are they mutually exclusive?

3. What is the *sample space*?

4. What is the probability of drawing either a jack or a king?

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Example 3:
You have a well-shuffled deck of cards (52 cards, four suits: diamonds (♦), spades (♠), hearts (♥) and clubs (♣)). Drawing one card. What is the probability of drawing either a jack or a king?

1. What are the events we are looking for?
   Drawing a jack, drawing a king
2. Are they mutually exclusive?
   Yes they are, because jack≠king,
   i.e. one card cannot be a jack and a king at the same time

3. What is the *sample space*?
   52 card faces: 2♦, 2♥, 2♣, ..., Ace♠
   |sample space | = 52

4. What is the probability of drawing either a jack or a king?

   P(jack or king) = P(jack) + P(king) = $\left(\frac{4}{52}\right)+\left(\frac{4}{52}\right)=\frac{8}{52}=\frac{2}{13}\approx 0.15$

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Example 4:
Laura is playing Monopoly. On her next move she needs to throw a sum bigger than 8 on two dice in order to land on her own property and pass Go. What is the probability of getting more than 8?

1. What are the events we are looking for?

2. Are they mutually exclusive?

3. What is the *sample space*?

4. What is the probability of getting more than 8?

Some Probability Rules – Compound Events

Example 4:
Laura is playing Monopoly. On her next move she needs to throw a sum bigger than 8 on two dice in order to land on her own property and pass Go. What is the probability of getting more than 8?

1. What are the events we are looking for?
   Getting 9, 10, 11 or 12 on two dice

2. Are they mutually exclusive?
   Yes. We cannot get, for example, 8 and 9 at the same time on two dice.

3. What is the *sample space*?
   (1,1),(1,2),(1,3),...,(6,5),(6,6)
   |sample space | = 36 (=6 × 6)

4. What is the probability of getting more than 8?
   P(more than 8)=P(9 or 10 or 11 or 12) = P(9) + P(10) + P(11)+P(12) =

   So let's count how many ways are there to get 9, 10, 11, and 12:
   ways of getting 9: (4,5), (5,4), (3,6), (6,3) → 4
   ways of getting 10:(5,5), (4,6), (6,4) → 3
   ways of getting 11: (5,6), (6,5) → 2
   ways of getting 12: (6,6) → 1

$$= \left(\frac{4}{36}\right)+\left(\frac{3}{36}\right)+\left(\frac{2}{36}\right)+\left(\frac{1}{36}\right)=\frac{10}{36}=\frac{5}{18}\approx 0.28$$

Some Probability Rules – Compound Events

Example 5: ***Survey: Trips to Supermarket***
How many times do shoppers go to the supermarket each week?
The age group of customers with frequent visits can influence store inventory and marketing methods. The following table is based on information from *Trends in the United States*. The columns represent number of visits to the primary supermarket in an average week. The rows represent age distribution.

Age (years)	One Visit	Two Visits	Three Visits	Four Visits	Five Visits	6 or more Visits	raw total
18 – 24	**65**	**58**	**12**	**5**	**4**	**4**	**148**
25 – 39	**386**	**230**	**69**	**22**	**17**	**15**	**739**
40 – 49	**210**	**161**	**36**	**13**	**9**	**5**	**434**
50 – 64	**186**	**102**	**35**	**14**	**7**	**5**	**349**
65 and over	**115**	**69**	**18**	**12**	**7**	**3**	**224**
Column Total	**962**	**620**	**170**	**66**	**44**	**32**	**1894**

What is the probability that a customer chosen at random
(a) has been to the supermarket at least 2 times this past week?
(b) has been to the supermarket at least 2 times this past week, given that he or she is 25 to 39 years old?
(c) has been to the supermarket more than 3 times this past week?
(d) has been to the supermarket more than 3 times this past week, given that he or she is 65 or older?
(e) is 40 or older?
(f) is 40 or older, given that he or she has visited the supermarket 4 times this past week?
(g)Are the events age 25-39 years and visits more than once a week independent? Explain.

Some Probability Rules – Compound Events

Example 5: ***Survey: Trips to Supermarket***

Age (years)	One Visit	Two Visits	Three Visits	Four Visits	Five Visits	6 or more Visits	raw total
18 – 24	**65**	**58**	**12**	**5**	**4**	**4**	**148**
25 – 39	**386**	**230**	**69**	**22**	**17**	**15**	**739**
40 – 49	**210**	**161**	**36**	**13**	**9**	**5**	**434**
50 – 64	**186**	**102**	**35**	**14**	**7**	**5**	**349**
65 and over	**115**	**69**	**18**	**12**	**7**	**3**	**224**
Column Total	**962**	**620**	**170**	**66**	**44**	**32**	**1894**

What is the probability that a customer chosen at random

(a) has been to the supermarket at least 2 times this past week?

P(at least 2 times)=P(2 or 3 or 4 or 5 or 6 more times)=(mutually exclusive)=
p(2) + P(3) + P(4) +P(5)+P(6)=(620+170+66+44+32)/1890=932/1894 ≈ 0.49 or 49%

(b) has been to the supermarket at least 2 times this past week, given that he or she is 25 to 39 years old?

P(least 2 times this past week, given that he or she is 25 to 39 years old)=(mutually exclusive)=P(2,given 25-39) + P(3,given 25-39) + P(4,given 25-39) + P(5,given 25-39) + P(6,given 25-39) = (230+69+22+17+15)/739=353/739 ≈ 0.48 or 48%

(c) has been to the supermarket more than 3 times this past week?

P(more than 3 times) = P(4 or 5 or 6)=(66+44+32)/1894=142/1894 ≈ 0.07

Some Probability Rules – Compound Events

Example 5: ***Survey: Trips to Supermarket***

Age (years)	One Visit	Two Visits	Three Visits	Four Visits	Five Visits	6 or more Visits	raw total
18 – 24	**65**	**58**	**12**	**5**	**4**	**4**	**148**
25 – 39	**386**	**230**	**69**	**22**	**17**	**15**	**739**
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Column Total	**962**	**620**	**170**	**66**	**44**	**32**	**1894**

(d) has been to the supermarket more than 3 times this past week, given that he or she is 65 or older?

P(more than 3 times this past week, given that he or she is 65 or older) = (mutually exclusive) = P(4, given 65) + P(5, given 65) + P(6, given 65) = (12+7+3)/224 = 22/224 ≈ 0.1 or 10%

(e) is 40 or older?

P(40 or older) = P(40-49 or 50-64 or 65 and over)=(mutually exclusive) = P(40-49) + P(50-64) + P(65 and over) = (434+349+224)/1894=1007/1894 P ≈ 0.53 or 53%

(f) is 40 or older, given that he or she has visited the supermarket 4 times this past week?
P(40 or older, given that he or she has visited the supermarket 4 times this past week) = (mutually exclusive) = P(40-49, given 4 times) + P(50-64,given 4 times) + P(65 and more, given 4 times) = (13+14+12)/66=39/66 ≈ 0.59 or 59%

Some Probability Rules – Compound Events

Example 5: ***Survey: Trips to Supermarket***

Age (years)	One Visit	Two Visits	Three Visits	Four Visits	Five Visits	6 or more Visits	raw total
18 – 24	**65**	**58**	**12**	**5**	**4**	**4**	**148**
25 – 39	**386**	**230**	**69**	**22**	**17**	**15**	**739**
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Column Total	**962**	**620**	**170**	**66**	**44**	**32**	**1894**

(g) Are the events age 25-39 years and visits more than once a week independent? Explain.

Independent.