

## 7.3 Areas Under Any Normal Curve

### **Example1:**

Let  $x$  have a normal probability distribution with  $\mu = 4$  and  $\sigma = 2$ . Find the probability that  $x$  value selected at random from the distribution is between 2 and 7.

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$$\mu = 4, \sigma = 2$$

$$\text{Find } P(2 \leq x \leq 7)$$

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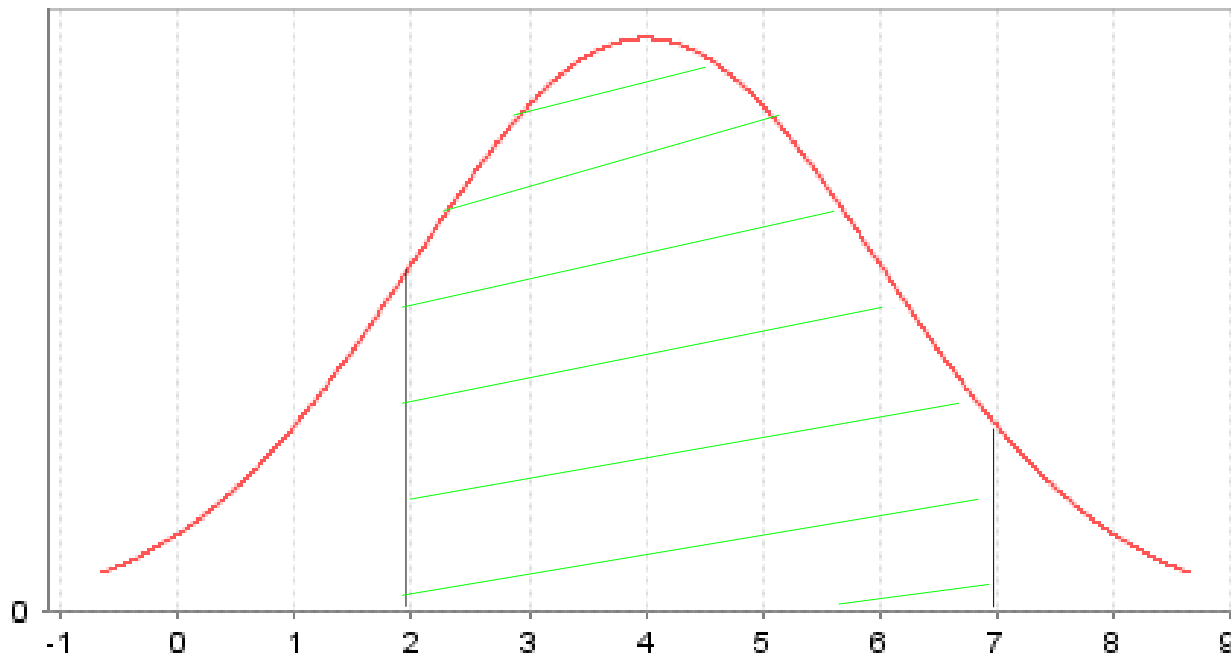
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$$\mu = 4, \sigma = 2$$

Find  $P(2 \leq x \leq 7)$

$P(2 \leq x \leq 7) =$  (area under the normal curve for  $x$  between 2 and 7)

Normal Curve for  $x$  distribution with  $\mu = 4, \sigma = 2$



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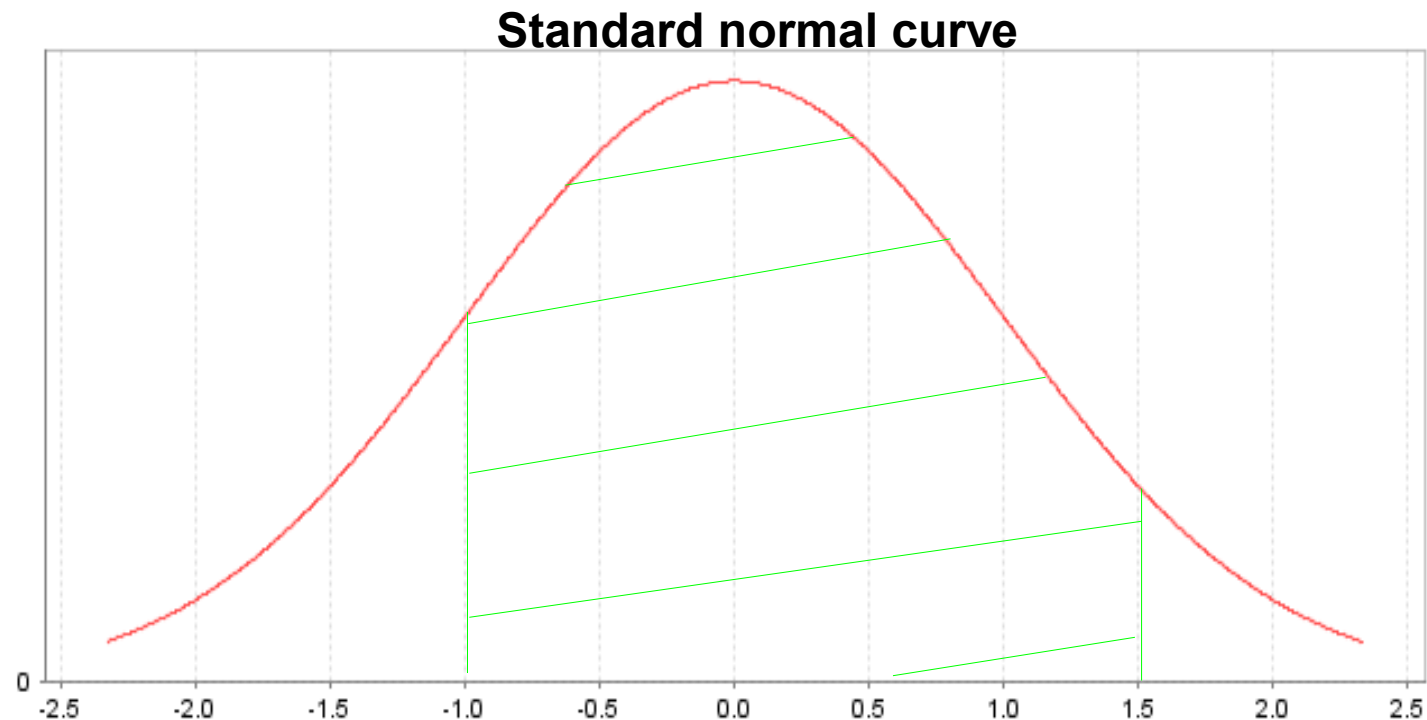
$$\mu = 4, \sigma = 2$$

$$\text{Find } P(2 \leq x \leq 7)$$

$$P(2 \leq x \leq 7) = (\text{area under the normal curve for } x \text{ between 2 and 7})$$

Let's convert the interval (from raw values to  $z$  values):

$$\begin{array}{ccc} 2 & \leq & x & \leq & 7 \\ \frac{(2-4)}{2} & & \frac{(x-\mu)}{\sigma} & & \frac{(7-4)}{2} \\ -1 & \leq & z & \leq & 1.5 \end{array}$$



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$$\text{Find } P(2 \leq x \leq 7)$$

$$\begin{aligned} P(2 \leq x \leq 7) &= (\text{area under the } \underline{\text{normal}} \text{ curve for } x \text{ between 2 and 7}) = \\ &= P(-1 \leq z \leq 1.5) = (\text{area under the } \underline{\text{standard normal}} \text{ curve for } z \text{ between -1 and 1.5}) = \\ &= (\text{area to the left of 1.5}) - (\text{area to the left of -1}) = P(z \leq 1.5) - P(z \leq -1) = (\text{use Table 3}) \\ &= 0.9332 - 0.1587 = 0.7745 \end{aligned}$$

$$P(2 \leq x \leq 7) = 0.7745$$

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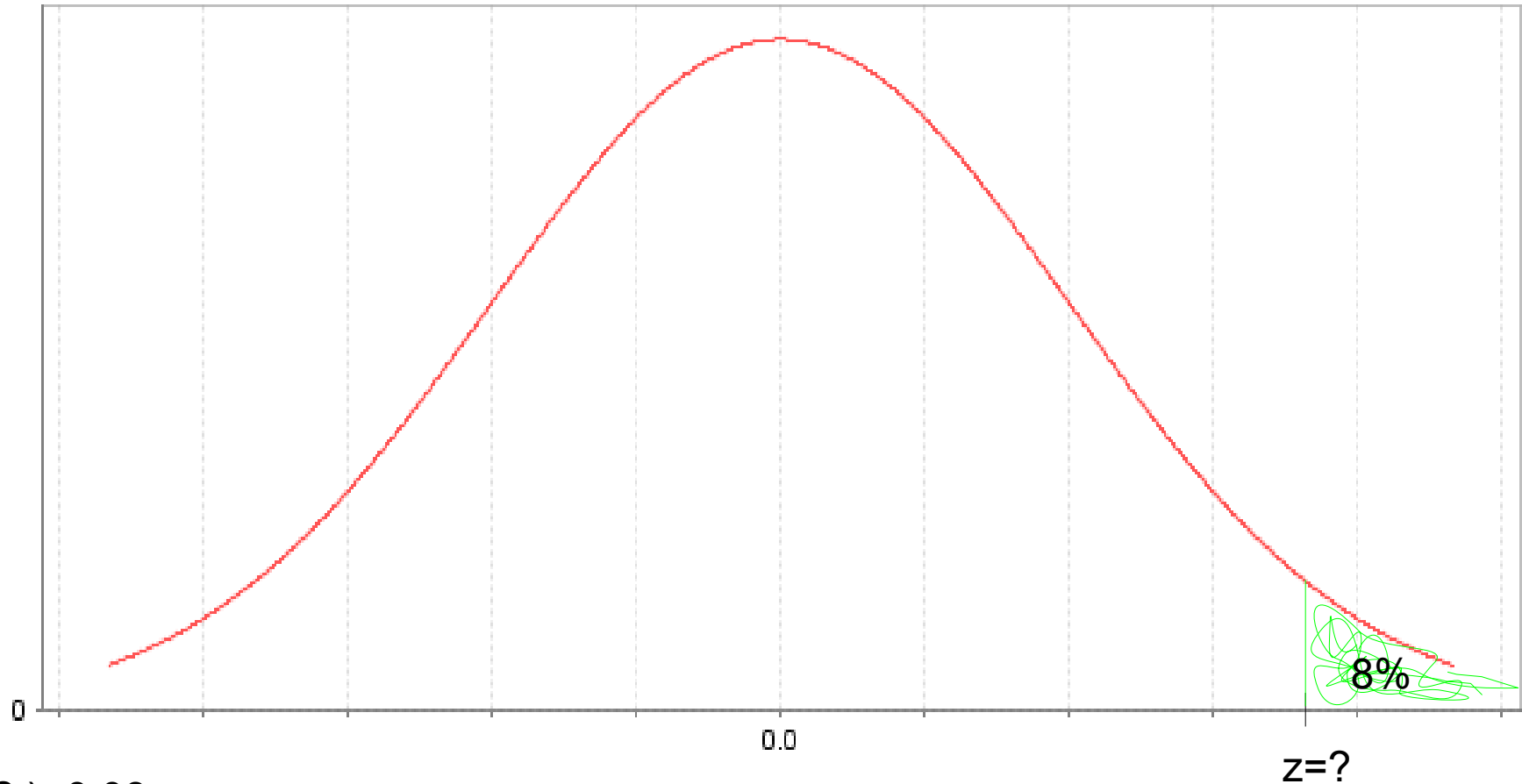
### **Example2 ( p. 277/15 ):**

Find the z-value such that 8% of the standard normal curve lies to the right of z.  
Sketch the area described.

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Sketch the area described.



$$P(z \geq ?) = 0.08$$

$P(z \leq ?) = 1 - 0.08 = 0.92$  Let's find the area .9200 in the Table 3 (or the closest to it):  
- closest is .9207 (.9192 is farther). Thus  $z \approx 1.41$

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### **Example3 ( p. 278/25 ):**

Quick Start Company makes 12-volt car batteries. After many years of product testing, the company knows that the average life of a Quick Start battery is normally distributed, with a mean of 45 months and a standard deviation of 8 months.

- (a) If Quick Start guarantees a full refund on any battery that fails within the 36-month period after purchase, what percentage of its batteries will the company expect to replace?
- (b) If Quick Start does not want to make refunds for more than 10% of its batteries under the full-refund guarantee policy, for how long should the company guarantee the batteries (to the nearest tenth)?

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Summary:  $\mu = 45$ ,  $\sigma = 8$

(a) If Quick Start guarantees a full refund on any battery that fails within the 36-month period after purchase, what percentage of its batteries will the company expect to replace?

$x=36$ , find  $P( x \leq 36 )$

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Summary:  $\mu = 45$ ,  $\sigma = 8$

(a) If Quick Start guarantees a full refund on any battery that fails within the 36-month period after purchase, what percentage of its batteries will the company expect to replace?

$$x=36, \text{ find } P( x \leq 36 ) = P( z \leq \frac{(36-45)}{8} ) = P( z \leq -1.12 ) = (\text{Table 3}) = .1314$$

13% of the batteries the company should expect to replace.

$$*: \frac{(36-45)}{8} = -1.125,$$

Table 3 has areas for z values with only two decimal places, thus we need either to round off -1.125 or to do something else. We decide to take the z-value with larger area.

(area to the left of -1.12) is more than (the area to the left of -1.13). Thus let's take  $P( z \leq -1.12 )$

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Summary:  $\mu = 45$ ,  $\sigma = 8$

(b) If Quick Start does not want to make refunds for more than 10% of its batteries under the full-refund guarantee policy, for how long should the company guarantee the batteries (to the nearest tenth)?

area=.10, find z value for that area, then find the raw score (x) for that z.

$$P( z \leq ? ) = .1000$$

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area=.10, find z value for that area, then find the raw score (x) for that z.

$$P( z \leq ? ) = .1000$$

Look into Table 3 (find the area closest to 0.1000):

.1003 is the closest (.0985 is farther)

z value for area 0.1003 is -1.28 (in standard units)

$$x \text{ value} = -1.28 * 8 + 45 = 34.76 \text{ (months)}$$

Guarantee the batteries for 35 months.