8.1 Estimating *μ when σ is known*

Some statistical terms:
population – a set of all measurements (or counts) (existing or conceptual)
sample – is a subset of measurements from the population
random samples – the most important samples for our purposes
a parameter – is a numerical descriptive measure of a population
a statistic – is a numerical descriptive measure of a sample

measure	statistic	parameter
mean	$\bar{x}$	μ
variance	s^2	σ^2
standard deviation	s	σ

We use statistic (such as $\bar{x}$, s, ...) to make inferences about corresponding population parameters (μ, σ, ...).

The principal types of inferences we'll make:
(1) estimate the value of a population parameter
(2) formulate a decision about the value of a population parameter

8.1 Estimating μ when σ is known

Assumptions about the random variable x:
(1) we have a simple random sample of size n drawn from a population of x values

(2) The value of σ is known

! If the value of σ is not known, but we have a large sample ($n \geq 30$),
then $\sigma \approx s$ is a good estimate for most practical purposes.

(3) If the x distribution is normal, then our methods work for any sample size n

If x has unknown distribution, then we require a sample size $n \geq 30$.
(although sometimes large samples may be necessary)

An estimate of a population parameter given by a single number is called a point estimate of that parameter.

We'll use $\bar{x}$ as a point estimate for μ,
also s is a point estimate for σ (for large samples)

Even with a large random sample the value of $\bar{x}$ usually is not exactly equal to μ.
The magnitude of difference between $\bar{x}$ and μ is called margin of error ($|\bar{x}-\mu|$)

Reliability of the an estimate is measured by the confidence level *c.*
c is usually given by a number between 0 and 1 ($0 < c < 1$), closer to 1.

8.1 Estimating μ when σ is known

We'll be using confidence level c to find an interval for μ.

What do we have:

- x – is a random variable appropriate for our application.
- a simple random sample (of size n) of x values from the population (from which we can compute $\bar{x}$ or maybe we already given it).
- the value of σ (if not then use s as an estimate if $n \geq 30$).

If x has a normal distribution, then the method works for any sample size n. If not, then the sample size n should be ≥ 30.

- confidence level c.

Steps to perform:

1. Find z_c (the critical value for the confidence level c) - use Table 3(b)

2. Find the maximal margin of error $E = z_c \frac{\sigma}{\sqrt{n}}$

3. The c confidence interval for μ is: $\bar{x} - E < \mu < \bar{x} + E$

When σ is unknown, but $n \geq 30$, use s

idea: $P(\bar{x} - z_c \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z_c \frac{\sigma}{\sqrt{n}}) = c$

8.1 Estimating *μ* when *σ* is known

Sometimes we want to set in advance the *confidence level* and the *maximum margin of error E* we want for our project.
And based on that determine the *sample size* (that we should use for our project).

How to find the sample size *n* for estimating *μ* when *σ* is known:

assume that the distribution of sample means $\bar{x}$ is approximately normal, then

$$n = \left(\frac{(z_c \sigma)}{E}\right)^2$$

E – specified maximal error of estimate
σ – population standard deviation
z_c – critical value from the normal distribution for the desired confidence level c

! If *n* is not a whole number, **increase** it to the next whole number.
Note that *n* is the minimal size for a specified confidence level and maximal error of estimate *E*

8.1 Estimating μ *when* σ *is known*

Example: page 324/1 *Zoology: Hummingbirds*
Allen's hummingbird has been studied by zoologist Bill Alther. A small group of 15 Allen's hummingbirds has been under study in Arizona. The average weight for these birds is $\bar{x}=3.15$ *gm*. Based on the previous studies, we can assume that the weights of Allen's hummingbirds have a normal distribution with $\sigma=0.33$ *gm*.

(a) Find an *80%* confidence interval for the average weights of Allen's hummingbirds in the study region. What is the margin of error?

(b) What conditions are necessary for your calculations?

(c) Give a brief interpretation of your results in the context of this problem.

(d) Find the sample size necessary for an *80%* confidence level with a maximal error of estimate $E = 0.08$ for the mean weights of the hummingbirds.

8.1 Estimating μ when σ is known

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(a) Find an *80%* confidence interval for the average weights of Allen's hummingbirds in the study region. What is the margin of error?

C=80%=0.8 Look into the table 3(b): $z_c=1.28$

$$E = z_c \frac{\sigma}{\sqrt{n}} = 1.28 \frac{0.33}{\sqrt{15}} \approx 0.11$$

$\bar{x} - E = 3.15 - 0.11 = 3.04$ $\qquad$ $\bar{x} + E = 3.15 + 0.11 = 3.26$

The 80% confidence interval for μ is: $3.04 < \mu < 3.26$

8.1 Estimating *μ when σ is known*

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The 80% confidence interval for μ is: $3.04 < \mu < 3.26$

(b) What conditions are necessary for your calculations?

Distribution of x values (the weighs of Allen's hummingbirds) is normal; *σ is known*

(c) Give a brief interpretation of your results in the context of this problem.

With 80% confidence we can say that the average weight of Allen's hummingbirds in Arizona is between 3.04 gramms and 3.26 gramms.

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(d) Find the sample size necessary for an *80%* confidence level with a maximal error of estimate $E = 0.08$ for the mean weights of the hummingbirds.

$c = 0.8$ using Table 3(b) and $z_c=1.28$; $E=0.08$; $\sigma=0.33$

$$n = \left(\frac{(z_c \sigma)}{E}\right)^2 = \left(\frac{(1.28*0.33)}{0.08}\right)^2 = 27.8784$$

- not a whole number

$n = 28$

i.e. In order to get a 80% confidence interval for the average weight of Allen's hummingbirds in Arizona with the maximal error of estimate 0.08 we need groups with at least 28 hummingbirds.