Measures of Variation

Example 1: page 93/1. *General Concepts: Variance, Standard Deviation*
Given the sample data

x : 23 17 15 30 25

(a) Find the range
(b) Verify that $\Sigma x=110$, and $\Sigma x^2=2568$
(c) Use the results of part (b) and appropriate computation formulas to compute the sample variance s^2 and sample standard deviation *s.*
(d) Use defining formulas to compute the sample variance s^2 and sample standard deviation *s.*
(e) Suppose the given data comprise the entire population of all x values. Compute the population variance σ^2 and population standard deviation σ.

(a) range = largest data value – smallest data value = 30-15=**15**
range = 15

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Given the sample data

***x* : 23 17 15 30 25**

(b) Verify that Σx=110

(d) Use **defining formulas** to compute the sample variance *s*^{2} and sample standard deviation *s.*

Let's compute the *sample mean*: $\bar{x} = \frac{\sum x}{n} = \frac{(23+17+15+30+25)}{5} = \frac{110}{5} = 22$

X	$X-\bar{X}$	$(X-\bar{X})^2$
23		
17		
15		
30		
25		
Σx=110		

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(d) Use **defining formulas** to compute the sample variance s^2 and sample standard deviation *s.*

Let's compute the *sample mean*: $\bar{x}$ = $\frac{\sum x}{n} = \frac{(23+17+15+30+25)}{5} = \frac{110}{5} = 22$

X	X-$\bar{X}$	$(X-\bar{X})^2$
23	23-22=1	$1^2=1$
17	17-22=-5	$(-5)^2=25$
15	15-22=-7	$(-7)^2=49$
30	30-22=8	$(8)^2=64$
25	25-22=3	$(3)^2=9$
Σx=110		$\Sigma(x-\bar{x})^2=1+25+49+64+9=148$

Thus *sample variance* s^2 = $\frac{\sum (x-\bar{x})^2}{(n-1)} = \frac{148}{4} = 37$,

and sample standard deviation is **s** = $\sqrt{37} \approx 6.083$

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Given the sample data

x : 23 17 15 30 25

(b) Verify that $\Sigma x=110$, and $\Sigma x^2=2568$

(c) Use the results of part (b) and appropriate **computation formulas** to compute the sample variance s^2 and sample standard deviation s.

X	X^2
23	529
17	289
15	225
30	900
25	625
$\Sigma x=110$	$\Sigma x^2=\Sigma(x^2)=2568$

Easier computation

sample variance

$$s^2=\frac{\left(\sum(x^2)-\left(\frac{(\sum x)^2}{n}\right)\right)}{(n-1)}=\frac{\left(2568-\frac{110^2}{5}\right)}{4}=\frac{\left(2568-\frac{12100}{5}\right)}{4}=\frac{(2568-2420)}{4}=\frac{148}{4}=37$$

sample standard deviation s = $\sqrt{37}\approx 6.083$

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Given the sample data

x : 23 17 15 30 25

(e) Suppose the given data comprise the entire population of all x values. Compute the population variance σ^2 and population stabdard deviation σ.

Let's compute the *population mean*: **μ =** $\frac{\sum x}{N} = \frac{(23+17+15+30+25)}{5} = \frac{110}{5} = 22$

X	X^2
23	529
17	289
15	225
30	900
25	625
Σx=110	$\Sigma x^2 = \Sigma(x^2) = 2568$

population standard deviation ***σ*** = $\sqrt{29.6} \approx 5.441$

population variance $\sigma^2 = \frac{(\sum(x^2) - (\frac{(\sum x)^2}{N}))}{N} = \frac{(2568 - \frac{110^2}{5})}{5} = \frac{(2568 - \frac{12100}{5})}{5} = \frac{(2568-2420)}{5} = \frac{148}{5} = 29.6$

Answers:

range = 15

sample mean $\bar{x} = 22$

sample variance $s^2=37$

sample standard deviation $s \approx 6.083$

population mean $\mu = 22$

population variance $\sigma^2 = 29.6$

poplation standard deviation $\sigma \approx 5.441$