22000 Algorithms, Summer 2004, CCNY CUNY

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Practice problems for Midterm Exam

1. Prove that the recurrence T(n) = T(n-1)+2T(n-2) is $\Omega(2^n)$;
   T(0) = 3, T(1) = 15 - boundary conditions

   answer:
   here the solution is needed. Solutions will be posted later.

2. Draw a recursion tree to determine a good assymptotic upper bound on the recurrence T(n) = $3T(\frac{n}{3}) + 4T(\frac{n}{4}) + n^2$

   answer: T(n) = $O(n^2 2^{\log_3 n})$

3. Is an array that is in reverse sorted order a heap (max-heap in terms of new book)?

   answer: Yes.

4. Suppose that the **for** loop header in line 9 of the Counting-sort procedure is rewritten as
   9 **for** j ← 1 to *length*[A]
   Show that the algorithm still works properly.

   answer:
   here the solution is needed. Solutions will be posted later.

5. What is the running time of Quicksort when all elements of array A have the same value?

   answer: T(n) = $O(n \lg n)$

6. For the set of keys $\{1, 5, 10, 13, 16, 19, 20, 24, 50\}$ draw binary search trees of heights 3,4,5,6,7

   answer: see 6.jpg (the link to file 6.jpg can be found on the **Notices** page).

7. Is there exists a tree with more than one node which is a heap (max-heap in terms of new book) and at the same time is a binary search tree ? (explain why not or give an example if it is exists)

   answer: Yes. example: a binary tree T (or an array A) where all nodes have the same **key** value (all elements of the array A have the same value)

8. consider the **searching problem**:
   Input: A sequence of n numbers A = $< a_1, \ldots, a_n >$ and a value v
   Output: An index i such that v = A[i] or the special value NIL if v doesn't appear in A.
   Write a pseudocode for **linear search**, which scans through the sequence,

looking for v. Give an assymptoticaly tight upper bound on its running time.

answer: the algorithm and its bound will be presented in the solutions.

9. Find an assymptotic upper bound on the summation

$$\sum_{k=0}^{n} \frac{k}{2^k}$$

by bounding the terms method.

answer: O(1)