

22000 Algorithms, Summer 2004, CCNY CUNY

Instructor: Natalia Novak

Practice problems for Midterm Exam

1. Prove that the recurrence  $T(n) = T(n-1) + 2T(n-2)$  is  $\Omega(2^n)$ ;  
 $T(0) = 3, T(1) = 15$  - boundary conditions
2. Draw a recursion tree to determine a good asymptotic upper bound on the recurrence  $T(n) = 3T(\frac{n}{3}) + 4T(\frac{n}{4}) + n^2$
3. Is an array that is in reverse sorted order a heap (max-heap in terms of new book)?
4. Suppose that the **for** loop header in line 9 of the Counting-sort procedure is rewritten as  
9 **for**  $j \leftarrow 1$  to  $length[A]$   
Show that the algorithm still works properly.
5. What is the running time of Quicksort when all elements of array A have the same value?
6. For the set of keys  $\{1, 5, 10, 13, 16, 19, 20, 24, 50\}$  draw binary search trees of heights 3,4,5,6,7
7. Is there exists a tree with more than one node which is a heap (max-heap in terms of new book) and at the same time is a binary search tree ?  
(explain why not or give an example if it is exists)
8. consider the **searching problem**:  
**Input:** A sequence of n numbers  $A = \langle a_1, \dots, a_n \rangle$  and a value  $v$   
**Output:** An index  $i$  such that  $v = A[i]$  or the special value NIL if  $v$  doesn't appear in A.  
Write a pseudocode for **linear search**, which scans through the sequence, looking for  $v$ . Give an asymptotically tight upper bound on its running time.
9. Find an asymptotic upper bound on the summation

$$\sum_{k=0}^n \frac{k}{2^k}$$

by bounding the terms method.